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Deep Reinforcement Learning

Natural gradients (TRPO, PPO)

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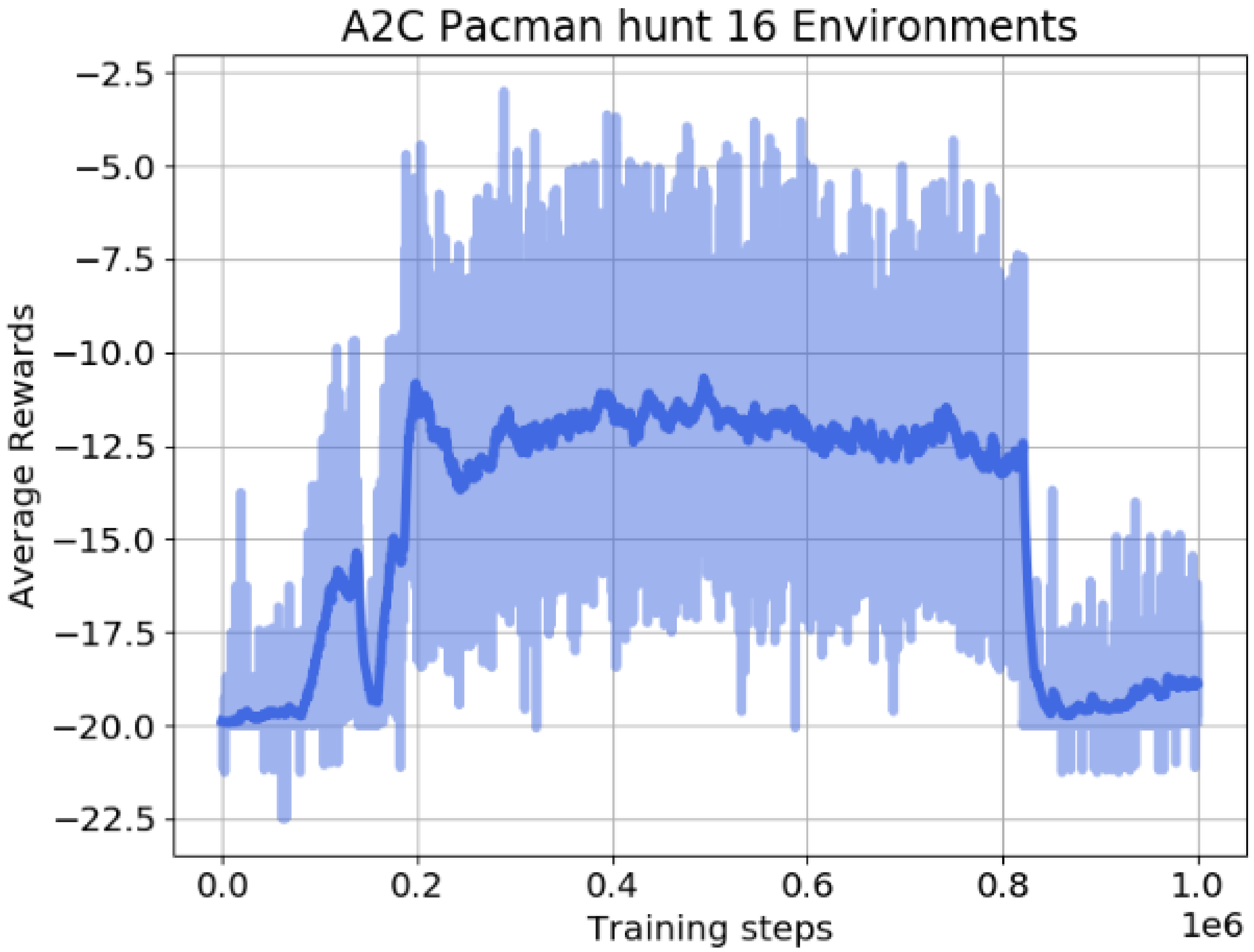
Trust regions and gradients

Source: https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9

- The policy gradient tells you in **which direction** of the parameter space θ the return is increasing the most.
- If you take too big a step in that direction, the new policy might become completely bad (**policy collapse**).
- Once the policy has collapsed, the new samples will all have a small return: the previous progress is lost.
- This is especially true when the parameter space has a **high curvature**, which is the case with deep NN.

Policy collapse

- Policy collapse is a huge problem in deep RL: the network starts learning correctly but suddenly collapses to a random agent.
- For on-policy methods, all progress is lost: the network has to relearn from scratch, as the new samples will be generated by a bad policy.



Oliver Lange (2019). Investigation of Model-Based Augmentation of Model-Free Reinforcement Learning Algorithms. MSc thesis, TU Chemnitz.

Trust regions and gradients

- **Trust region** optimization searches in the **neighborhood** of the current parameters θ which new value would maximize the return the most.
- This is a **constrained optimization** problem: we still want to maximize the return of the policy, but by keeping the policy as close as possible from its previous value.

Line search
(like gradient ascent)

Trust region

Source: https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9

Trust regions and gradients

- The size of the neighborhood determines the safety of the parameter change.
- In safe regions, we can take big steps. In dangerous regions, we have to take small steps.
- **Problem:** how can we estimate the safety of a parameter change?

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1 - TRPO: Trust Region Policy Optimization (skipped)

Trust Region Policy Optimization

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Schulman et al. (2015) Trust Region Policy Optimization. 1889–1897. http://proceedings.mlr.press/v37/schulman15.html.

2 - PPO: Proximal Policy Optimization

Proximal Policy Optimization Algorithms

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TRPO: Trust Region Policy Optimization

- We want to maximize the expected return of a policy π_θ, which is equivalent to maximizing the Q-value of every state-action pair visited by the policy:

$$\max_\theta \ \mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [Q^{\pi_\theta} (s, a)]$$

- Let's note θ_old the current value of the parameters of the policy π_{θ_old}.
- We search for a new policy π_θ with parameters θ which is always **better** than the current policy, i.e. where the Q-value of all actions is higher than with the current policy:

$$\max_\theta \ \mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [Q_\theta(s, a) - Q_{\theta_\text{old}} (s, a)]$$

- The quantity

$$A^{\pi_{\theta_\text{old}}} (s, a) = Q_\theta(s, a) - Q_{\theta_\text{old}} (s, a)$$

is the **advantage** of taking the action (s, a) and thereafter following π_θ, compared to following the current policy π_{θ_old}.

Kakade, S., and Langford, J. (2002). Approximately Optimal Approximate Reinforcement Learning. Proc. 19th International Conference on Machine Learning, 267–274.

TRPO: Trust Region Policy Optimization

- If we can estimate the advantages and maximize them, we can find a new policy π_θ with a higher return than the current one.

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}}(s, a)] = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [Q_\theta(s, a) - Q_{\theta_\text{old}}(s, a)]$$

- By definition, $\mathcal{L}(\theta_\text{old}) = 0$, so the policy maximizing $\mathcal{L}(\theta)$ has positive advantages and is at least better than π_{θ_old}.

$$\theta_\text{new} = \text{argmax}_\theta \; \mathcal{L}(\theta) \; \Rightarrow \; \mathcal{J}(\theta_\text{new}) \geq \mathcal{J}(\theta_\text{old})$$

- Maximizing the advantages ensures **monotonic improvement**: the new policy is always better than the previous one. Policy collapse is not possible!

TRPO: Trust Region Policy Optimization

- Let's take the unconstrained objective function of TRPO:

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}}(s, a)]$$

- In order to avoid sampling action from the **unknown** policy π_θ, we can use importance sampling with the current policy:

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_{\theta_\text{old}}} [\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a)]$$

with $\rho(s, a) = \dfrac{\pi_\theta(s, a)}{\pi_{\theta_\text{old}}(s, a)}$ being the **importance sampling weight**.

- But the importance sampling weight $\rho(s, a)$ introduces a lot of variance, worsening the sample complexity.
- Is there another way to make sure that π_θ is not very different from π_{θ_old}, therefore reducing the variance of the importance sampling weight?

TRPO: Trust Region Policy Optimization

- TRPO introduces a **constrained optimization** approach (Lagrange optimization):

$$\max_\theta \mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_{\theta_\text{old}}} [A^{\pi_{\theta_\text{old}}}(s, a)]$$

$$\text{such that: } D_\text{KL}(\pi_{\theta_\text{old}} || \pi_\theta) \leq \delta$$

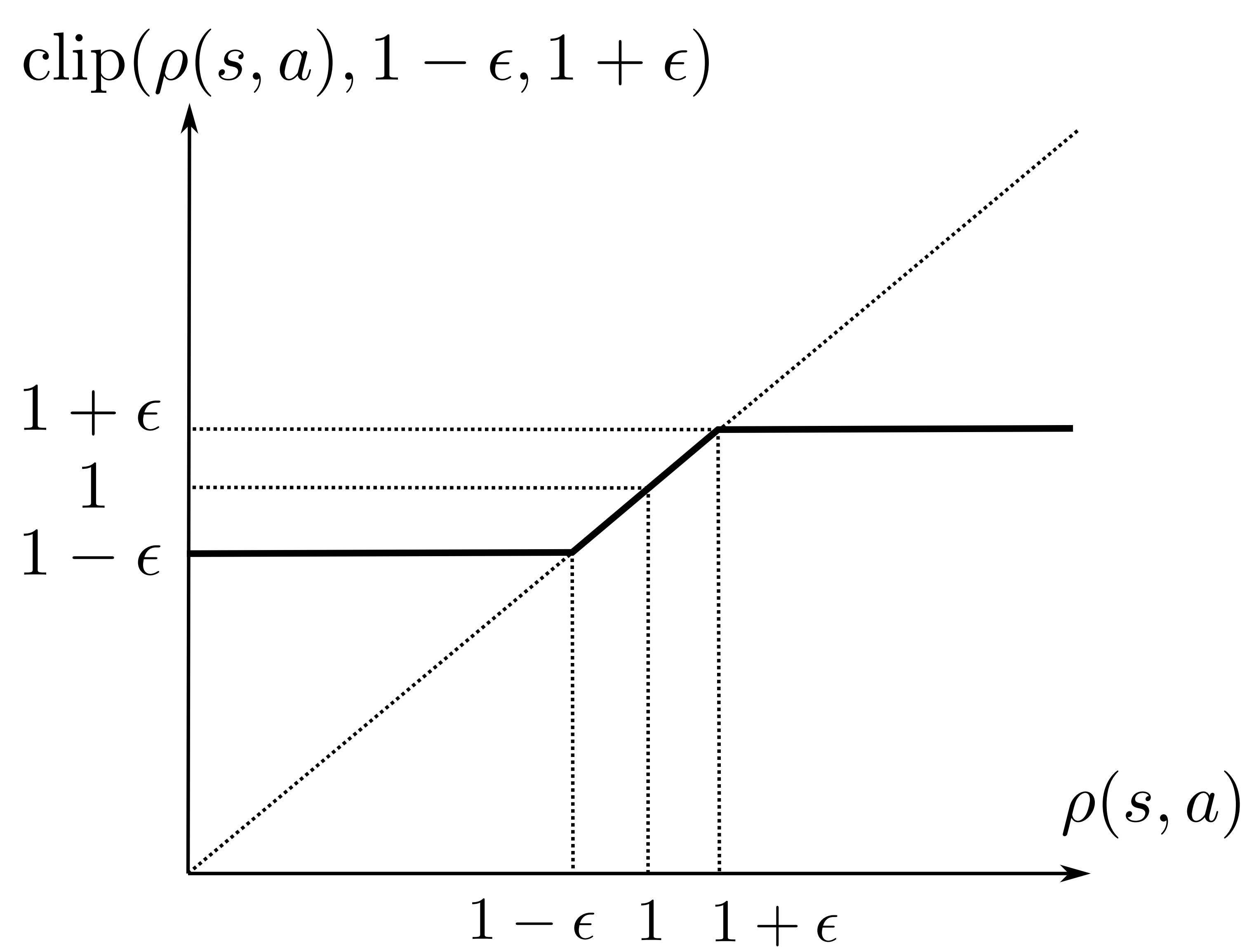
- The KL divergence between the distributions π_{θ_old} and π_θ must be below a threshold δ.
- We can neglect the importance sampling weight as long as the two policies are not very different (trust region).
- However, TRPO is very computationally expensive, as the constrained optimization problem involves conjugate gradients optimization, the Fisher Information matrix and natural gradients.
- The major interest of TRPO is the monotonic improvement guarantee.

PPO: Proximal Policy Optimization

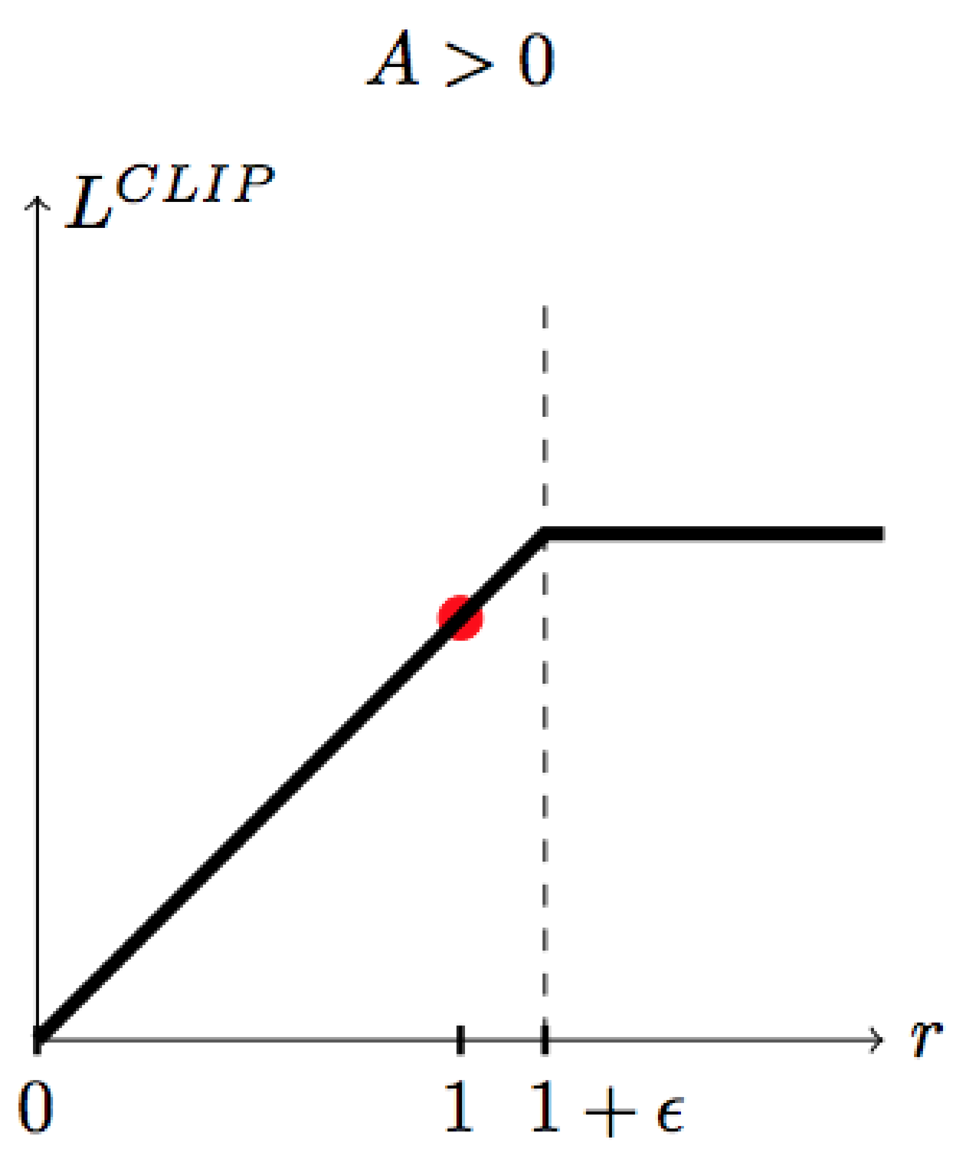
- The alternative solution introduced by PPO is simply to **clip** the importance sampling weight when it is too different from 1:

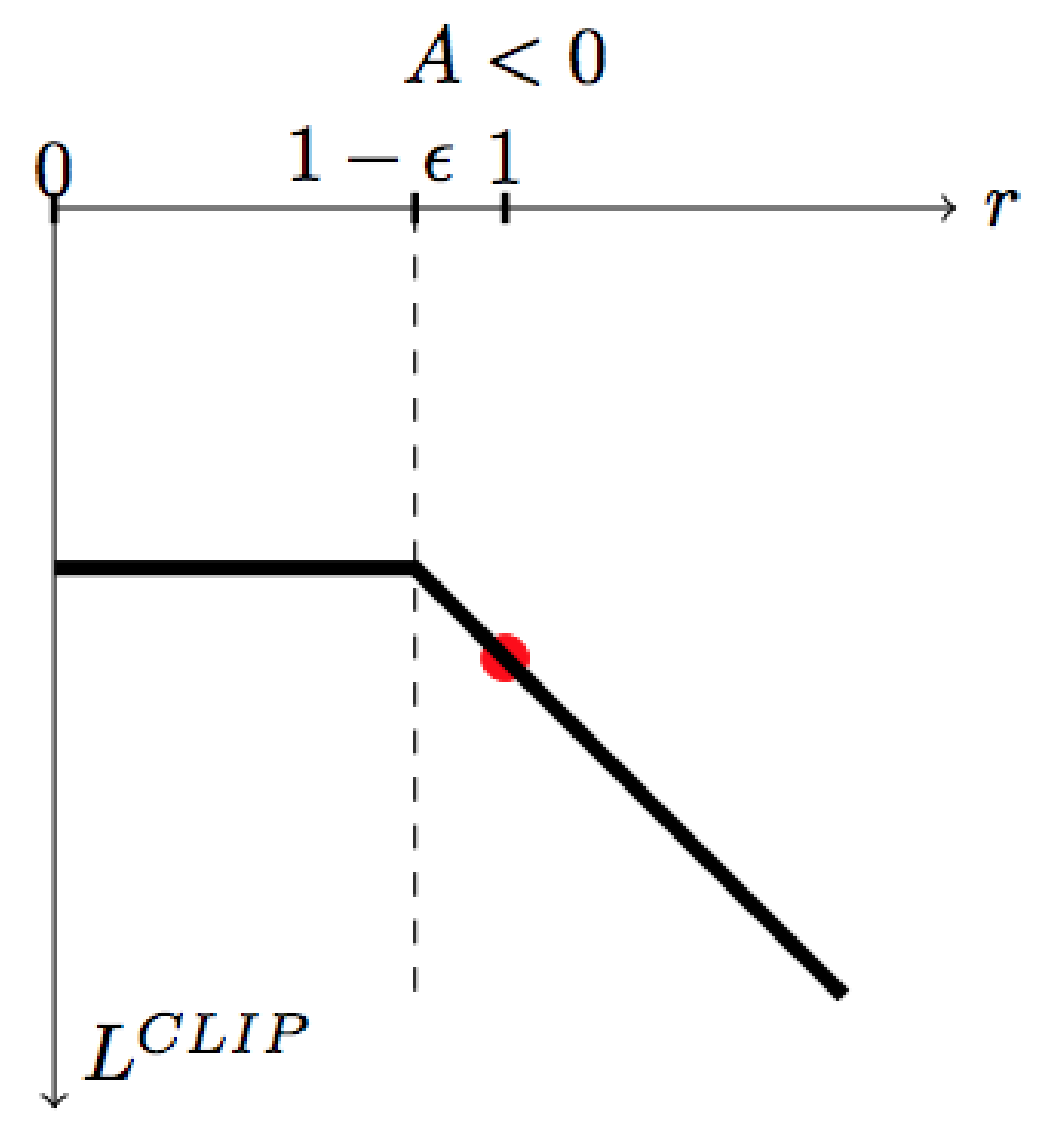
$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_{\theta_\text{old}}} [\min(\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a), \text{clip}(\rho(s, a), 1-\epsilon, 1+\epsilon) \, A^{\pi_{\theta_\text{old}}}(s, a))]$$

- For each sampled action (s, a), we use the minimum between:
    - the TRPO unconstrained objective with IS $\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a)$.
    - the same, but with the IS weight clipped between $1-\epsilon$ and $1+\epsilon$.



PPO: Proximal Policy Optimization



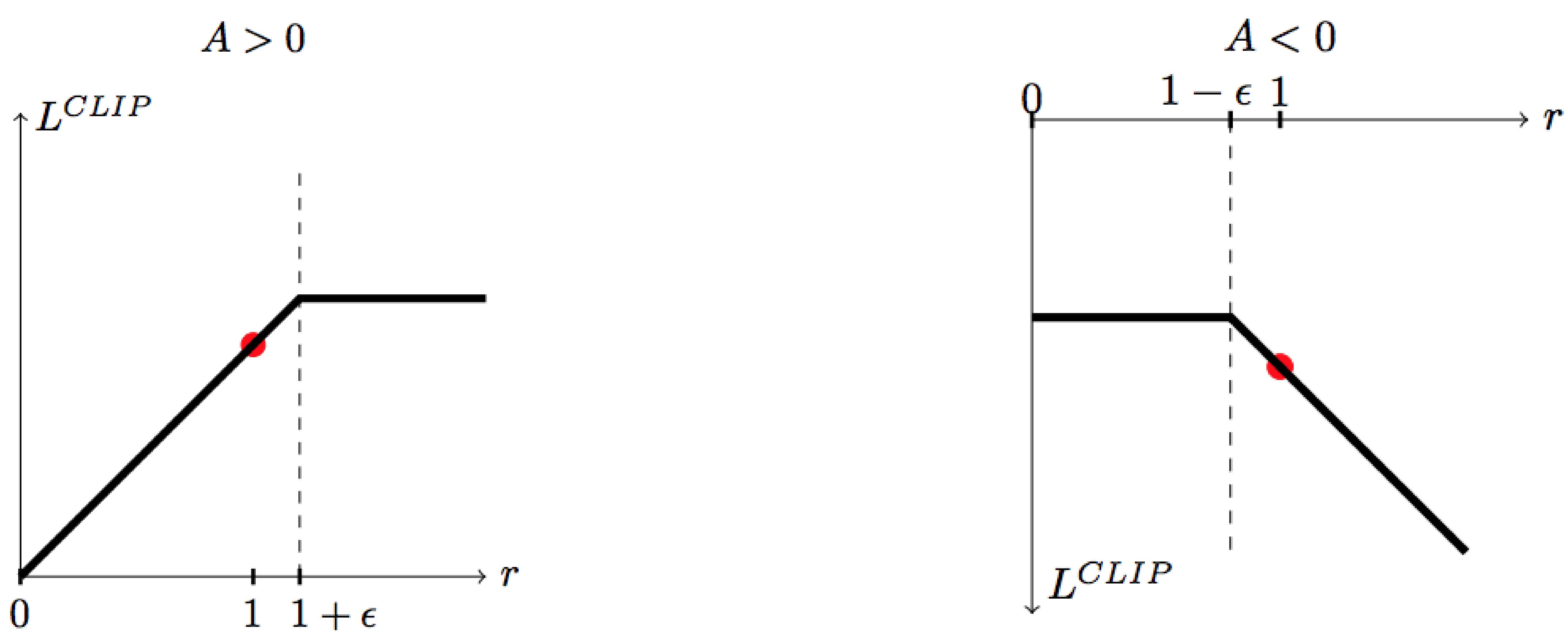


- If the advantage $A^{\pi_{\theta_\text{old}}}(s, a)$ is positive (better action than usual) and:
  - the IS is higher than $1+\epsilon$, we use $(1+\epsilon) \, A^{\pi_{\theta_\text{old}}}(s, a)$.
  - otherwise, we use $\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a)$.

- If the advantage $A^{\pi_{\theta_\text{old}}}(s, a)$ is negative (worse action than usual) and:
  - the IS is lower than $1-\epsilon$, we use $(1-\epsilon) \, A^{\pi_{\theta_\text{old}}}(s, a)$.
  - otherwise, we use $\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a)$.

Schulman et al. (2017) Proximal Policy Optimization Algorithms. arXiv:1707.06347.

PPO: Proximal Policy Optimization



- This avoids changing too much the policy between two updates:
  - Good actions ($A^{\pi_{\theta_\text{old}}}(s, a) > 0$) do not become much more likely than before.
  - Bad actions ($A^{\pi_{\theta_\text{old}}}(s, a) < 0$) do not become much less likely than before.

Schulman et al. (2017) Proximal Policy Optimization Algorithms. arXiv:1707.06347.

PPO: Proximal Policy Optimization

- The PPO **clipped objective** ensures than the importance sampling weight stays around one, so the new policy is not very different from the old one. It can learn from single transitions.

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_{\theta_\text{old}}} [\min(\rho(s, a) \, A^{\pi_{\theta_\text{old}}}(s, a), \text{clip}(\rho(s, a), 1 - \epsilon, 1 + \epsilon) \, A^{\pi_{\theta_\text{old}}}(s, a))]$$

- The advantage of an action can be learned using any advantage estimator, for example the **n-step advantage**:

$$A^{\pi_{\theta_\text{old}}}(s_t, a_t) = \sum_{k=0}^{n-1} \gamma^{k} \, r_{t+k+1} + \gamma^n \, V_\varphi(s_{t+n}) - V_\varphi(s_t)$$

- Most implementations use **Generalized Advantage Estimation** (GAE, Schulman et al., 2015).
- PPO is therefore an **actor-critic** method (as TRPO).
- PPO is **on-policy**: it collects samples using **distributed learning** (as A3C) and then applies several updates to the actor and critic.

Schulman et al. (2017) Proximal Policy Optimization Algorithms. arXiv:1707.06347.

PPO: Proximal Policy Optimization

- Initialize an actor π_θ and a critic V_φ with random weights.
- **while** not converged :
  - for N workers in parallel:
    - Collect T transitions using π_θ.
    - Compute the advantage $A_\varphi(s, a)$ of each transition using the critic V_φ.
  - for K epochs:
    - Sample M transitions $\mathcal{D}$ from the ones previously collected.
    - Train the actor to maximize the clipped surrogate objective.

$$\mathcal{L}(\theta) = \mathbb{E}_{s,a \sim \mathcal{D}}[\min(\rho(s, a) \, A_\varphi(s, a), \text{clip}(\rho(s, a), 1 - \epsilon, 1 + \epsilon) \, A_\varphi(s, a))]$$

    - Train the critic to minimize the advantage.

$$\mathcal{L}(\varphi) = \mathbb{E}_{s,a \sim \mathcal{D}}[(A_\varphi(s, a))^2]$$

PPO: Proximal Policy Optimization

- PPO is an **on-policy actor-critic** PG algorithm, using distributed learning.
- **Clipping** the importance sampling weight allows to avoid **policy collapse**, by staying in the **trust region** (the policy does not change much between two updates).
- The **monotonic improvement guarantee** is very important: the network will always find a (local) maximum of the returns.
- PPO is much less sensible to hyperparameters than DDPG (**brittleness**): works often out of the box with default settings.
- It does not necessitate complex optimization procedures like TRPO: first-order methods such as **SGD** work (easy to implement).
- The actor and the critic can **share weights** (unlike TRPO), allowing to work with pixel-based inputs, convolutional or recurrent layers.
- It can use **discrete or continuous action spaces**, although it is most efficient in the continuous case. Go-to method for robotics.
- Drawback: not very **sample efficient**.

PPO : Mujoco control

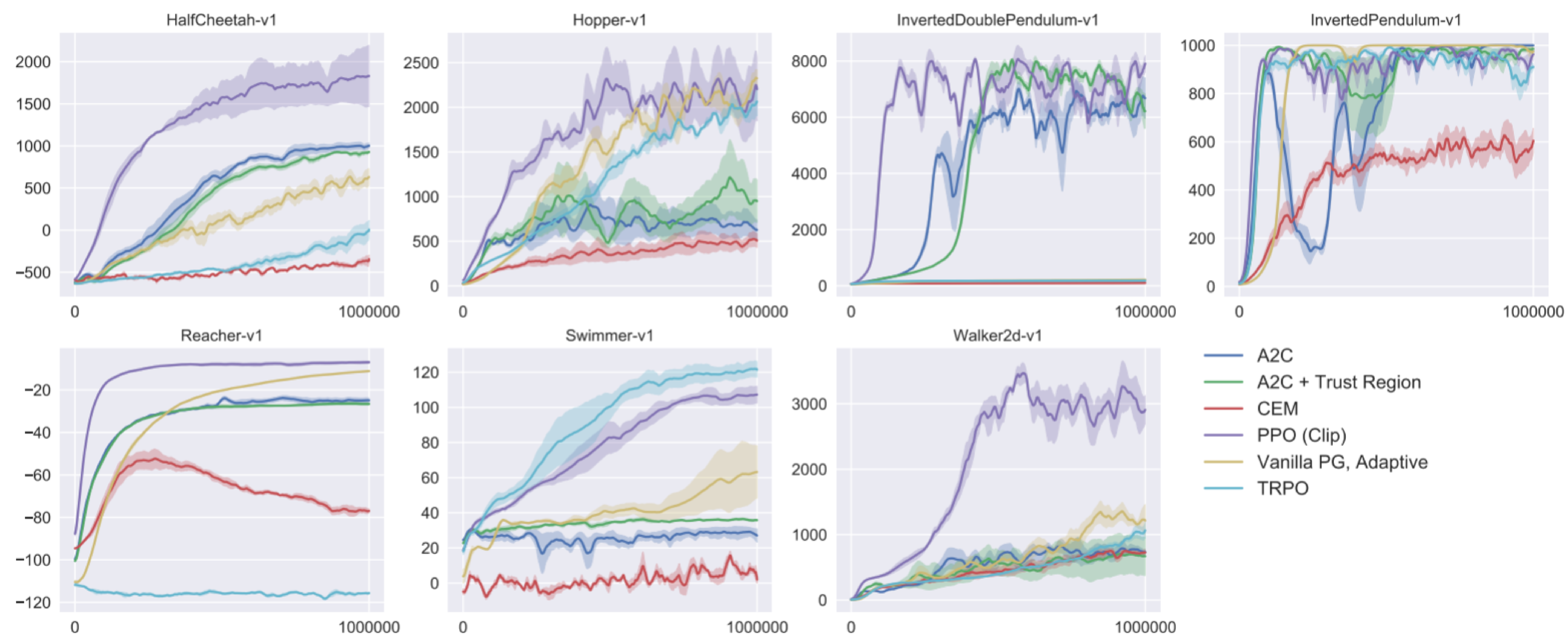
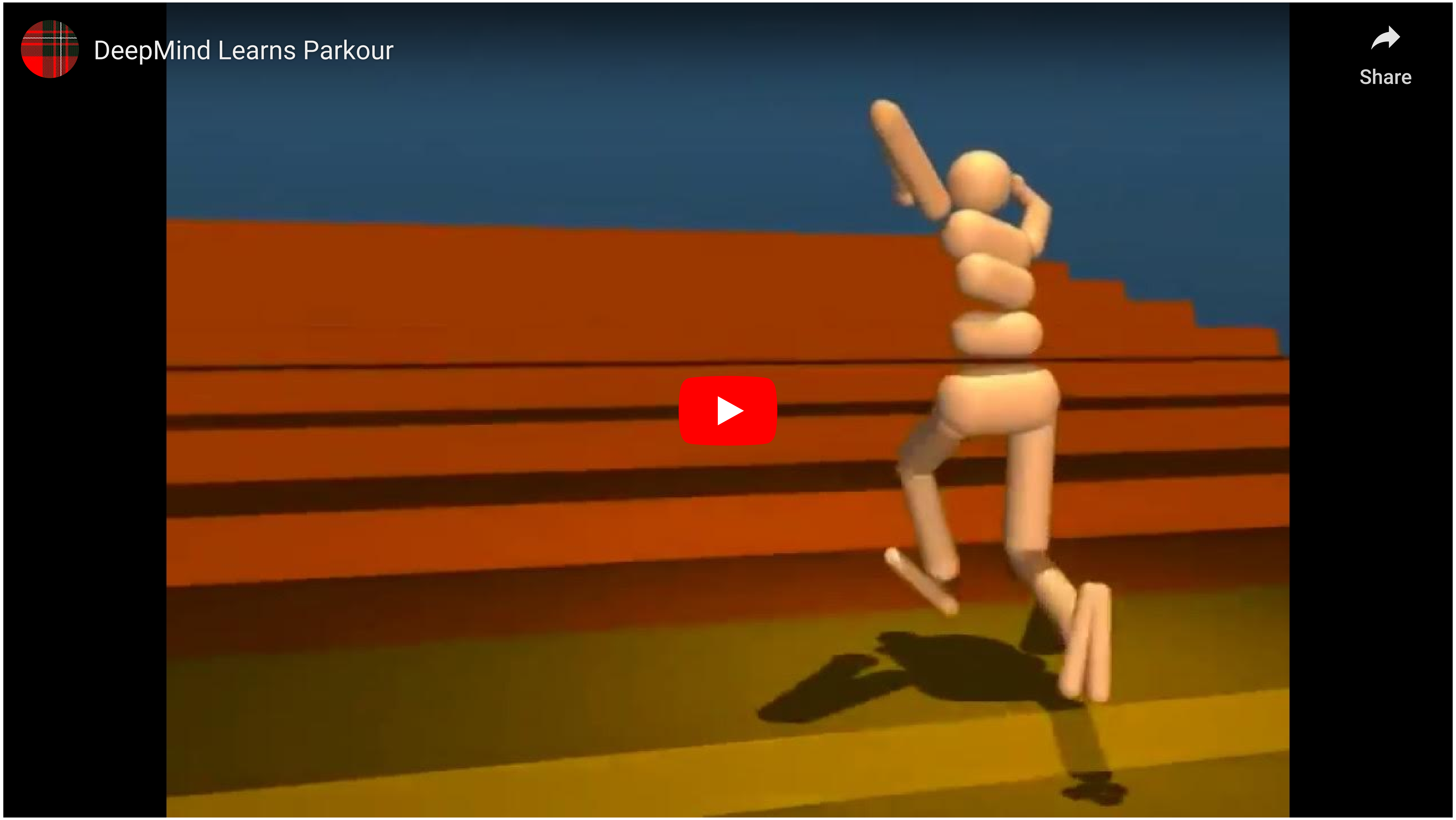


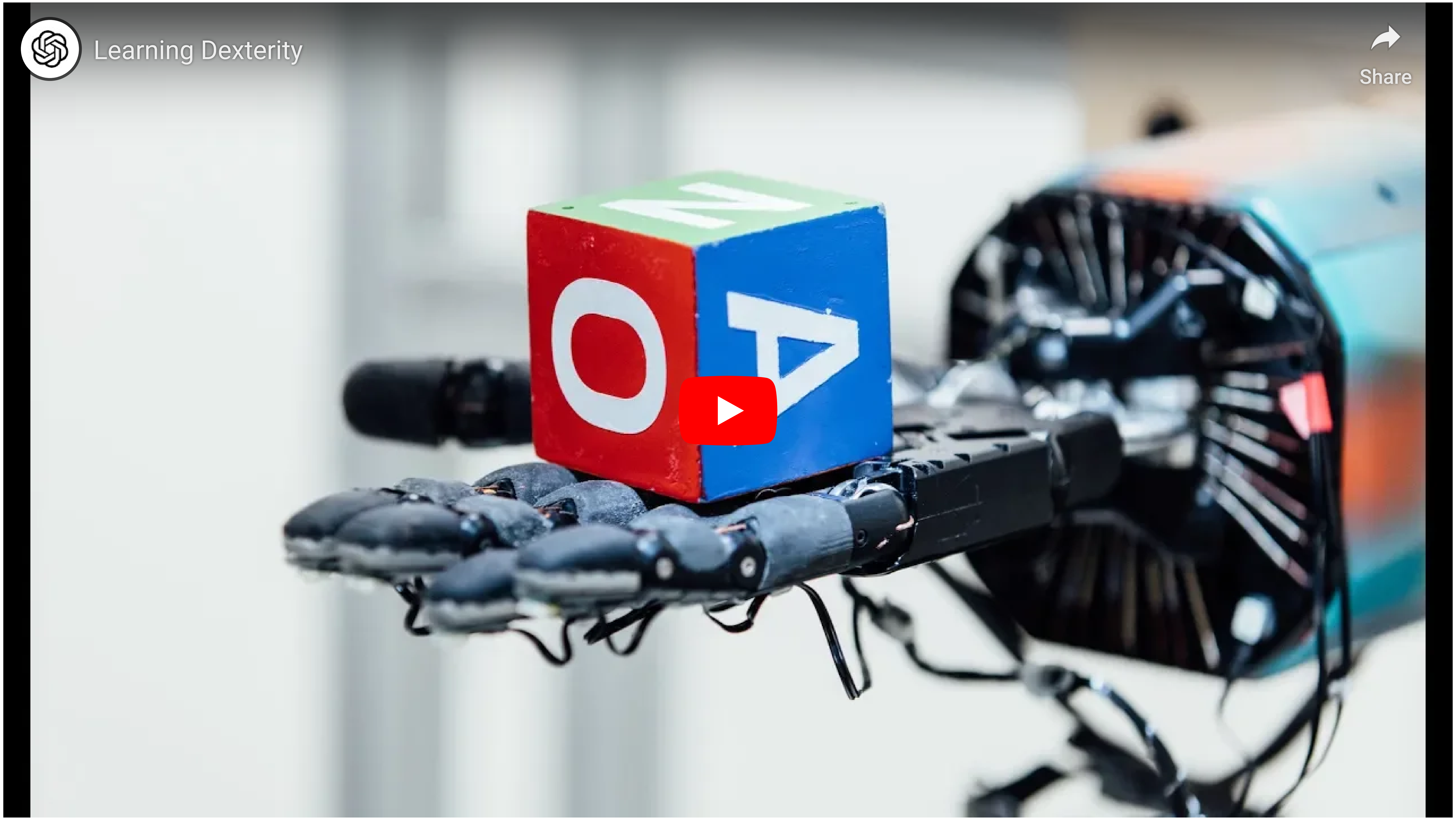
Figure 3: Comparison of several algorithms on several MuJoCo environments, training for one million timesteps.

PPO : Parkour

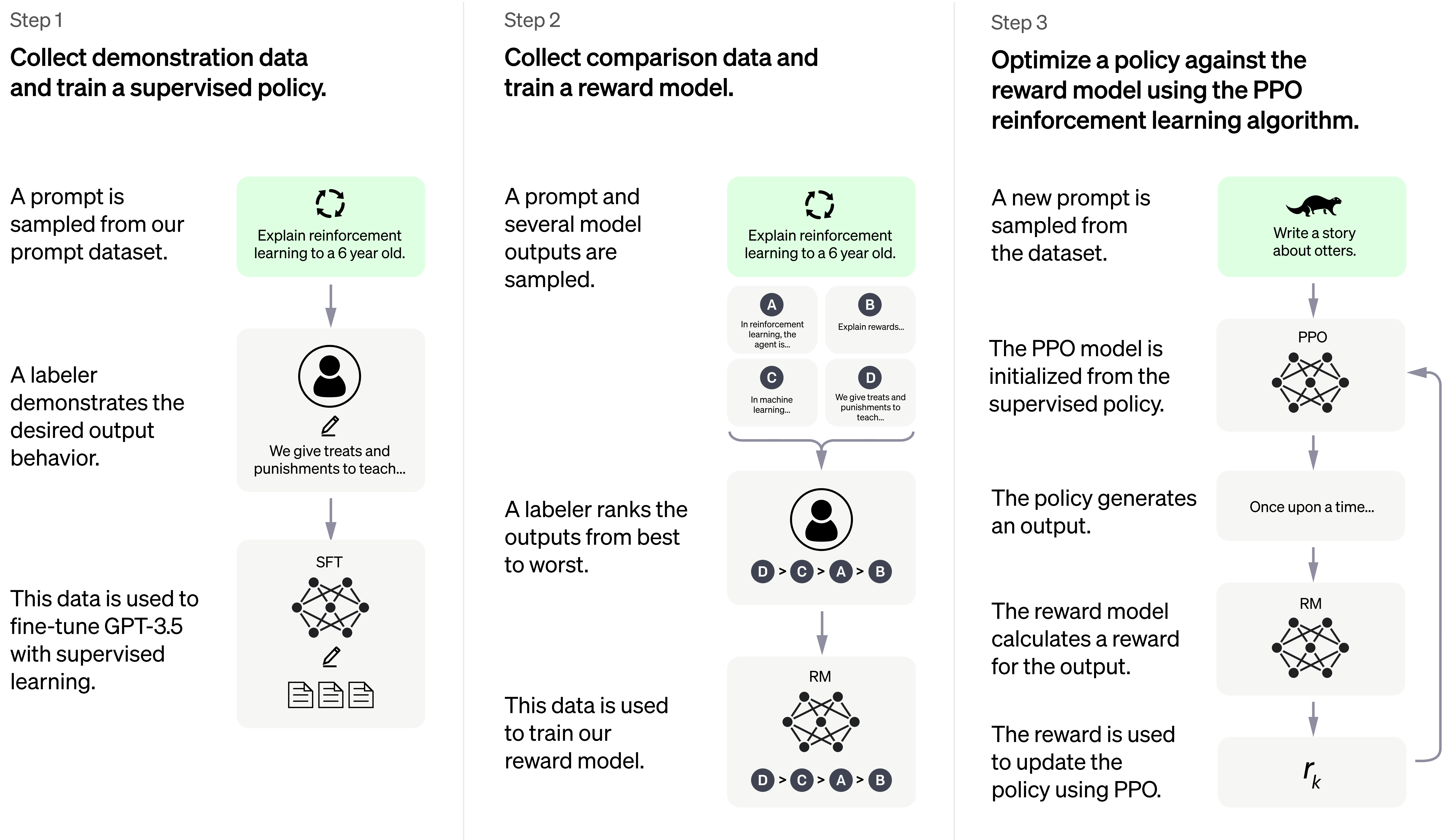


Check more robotic videos at: https://openai.com/blog/openai-baselines-ppo/

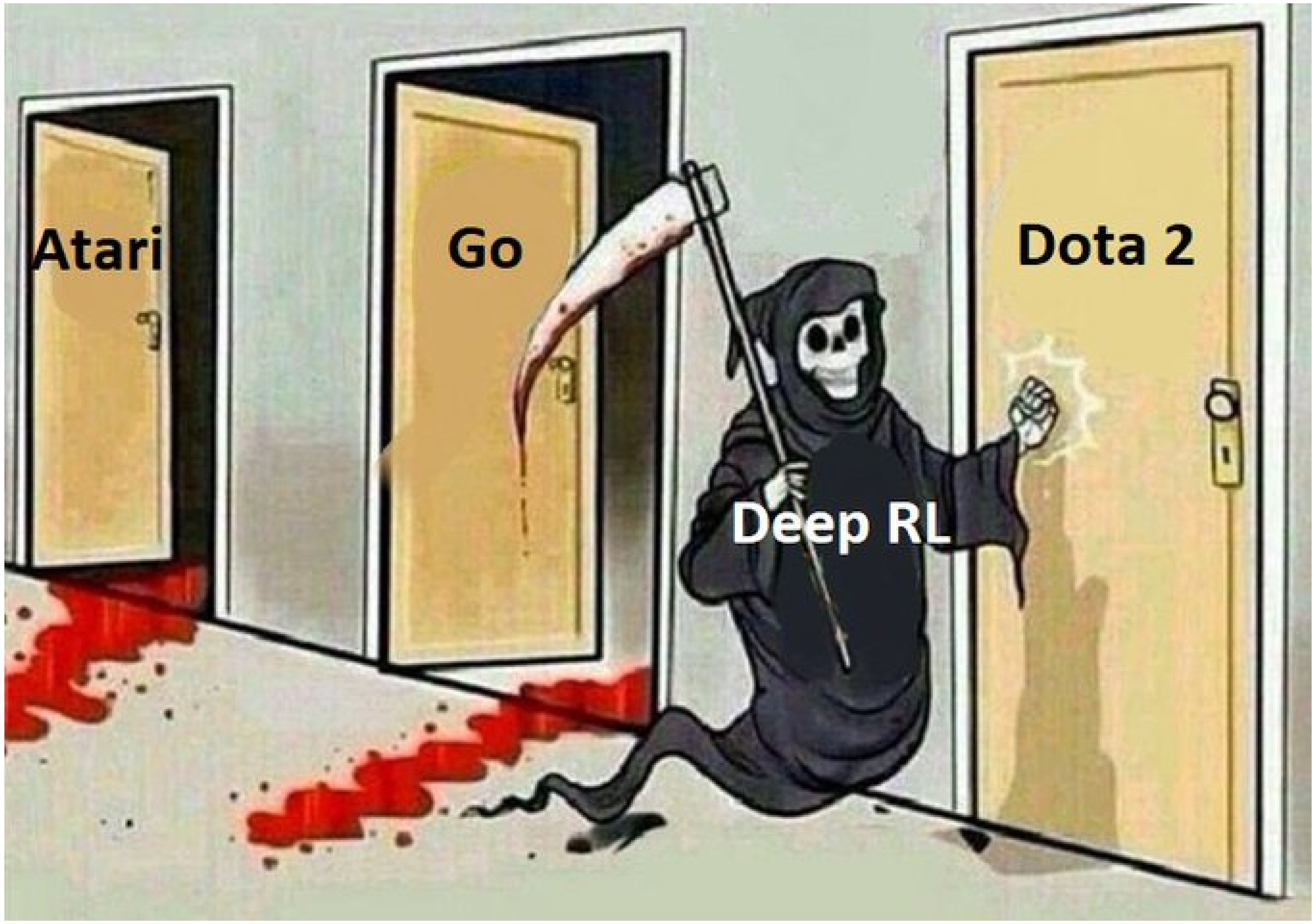
PPO: dexterity learning



PPO: Fine-tuning and alignment of ChatGPT

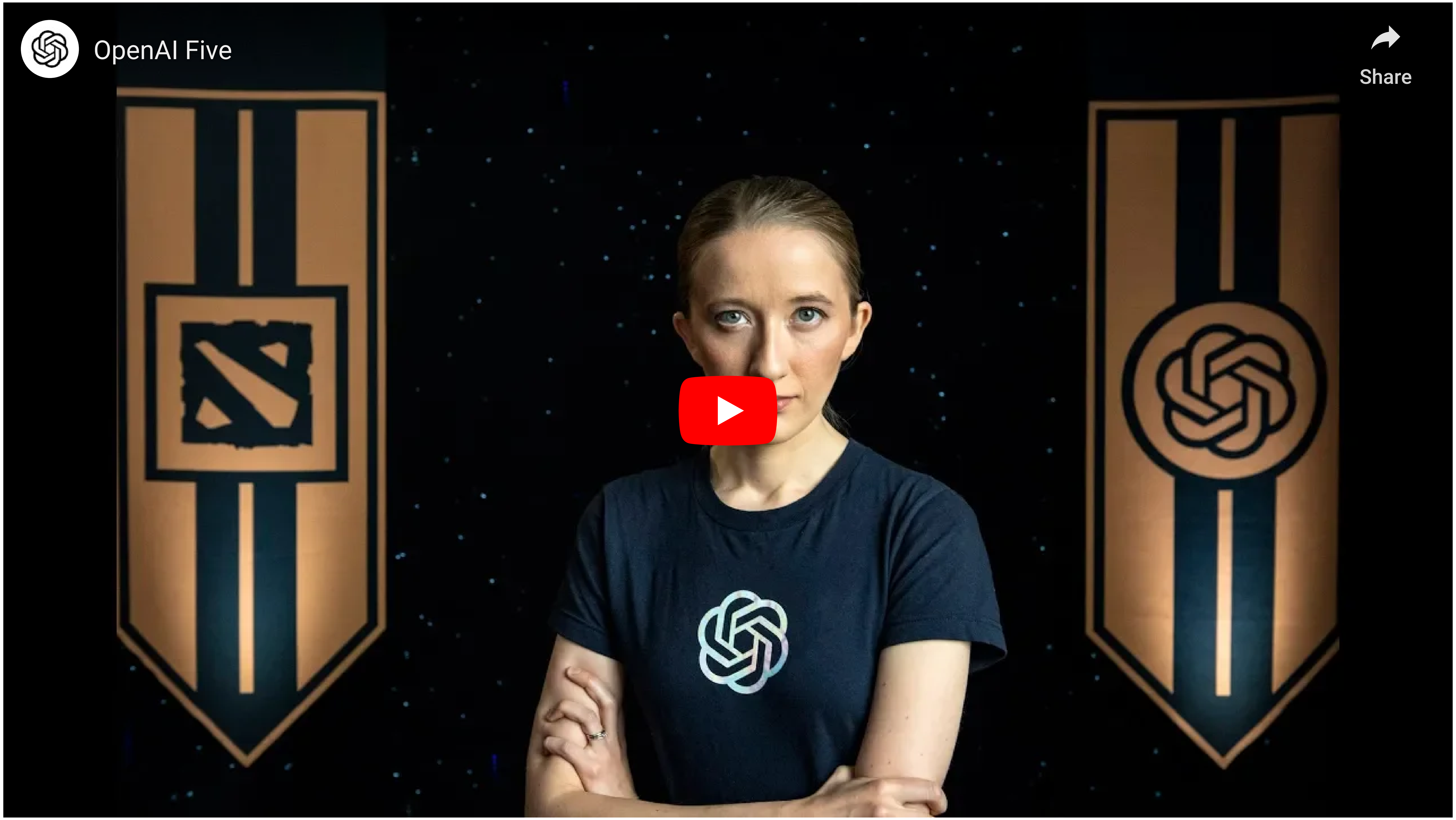


Source: https://openai.com/blog/chatgpt



twitter:ai-memes

3 - OpenAI Five: Dota 2



https://openai.com/projects/five/

Why is Dota 2 hard?

Long Time Horizons

- Most actions in Dota 2 have minor impact individually but contributed to the team's strategy.
- The game is about 20,000 moves long(compared to an average 40 moves of a chess match).

Partially Observed Stage

- At any given time, a team can only see a small area around them.
- Dota 2 strategies require making inference based on incomplete data.

Continuous Action Space

- Each hero is face with about 1000 actions each tick (compared to about 35 in chess)
- Actions can have completely different objectives such as targeting an enemy or improving the position on the ground

Continuous Observation Space

- The observation space in Dota 2 includes heterogenous components such as heroes, treesm buildings, trees, etc
- At any given point, the observations in a Dota 2 game can be quantified as 20,000 floating point numbers. The same quantifications for Chess and Go are about 70 and 400 numbers respectivey

Feature	Chess	Go	Dota 2
Total number of moves	40	150	20000
Number of possible actions	35	250	1000
Number of inputs	70	400	20000

https://openai.com/projects/five/

OpenAI Five: Dota 2

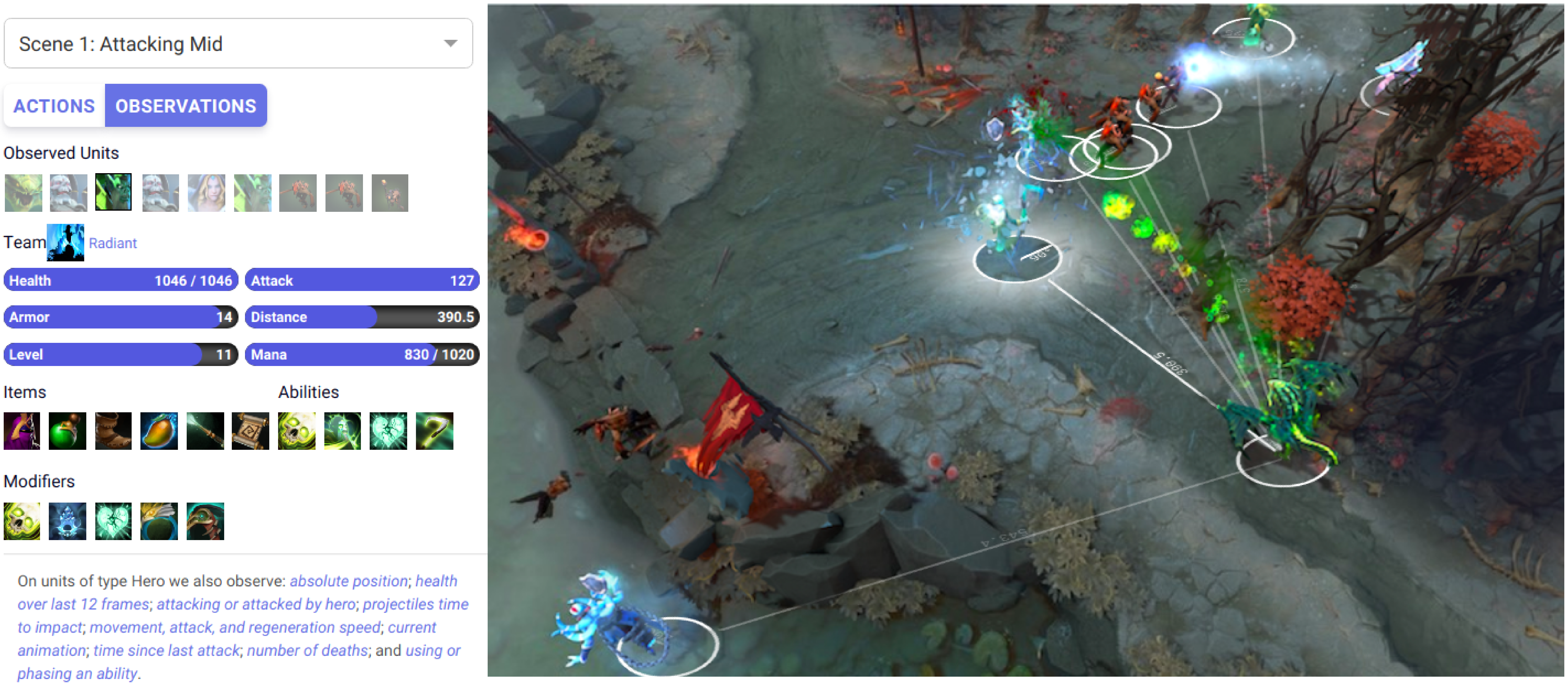
- OpenAI Five is composed of 5 PPO networks (one per player), using 128,000 CPUs and 256 V100 GPUs.

OpenAI Five: Dota 2

	OPENAI 1V1 BOT	OPENAI FIVE
CPUs	60,000 CPU cores on Azure	128,000 preemptible CPU cores on GCP
GPUs	256 K80 GPUs on Azure	256 P100 GPUs on GCP
Experience collected	~300 years per day	~180 years per day (~900 years per day counting each hero separately)
Size of observation	~3.3 kB	~36.8 kB
Observations per second of gameplay	10	7.5
Batch size	8,388,608 observations	1,048,576 observations
Batches per minute	~20	~60

https://openai.com/projects/five/

OpenAI Five: Dota 2



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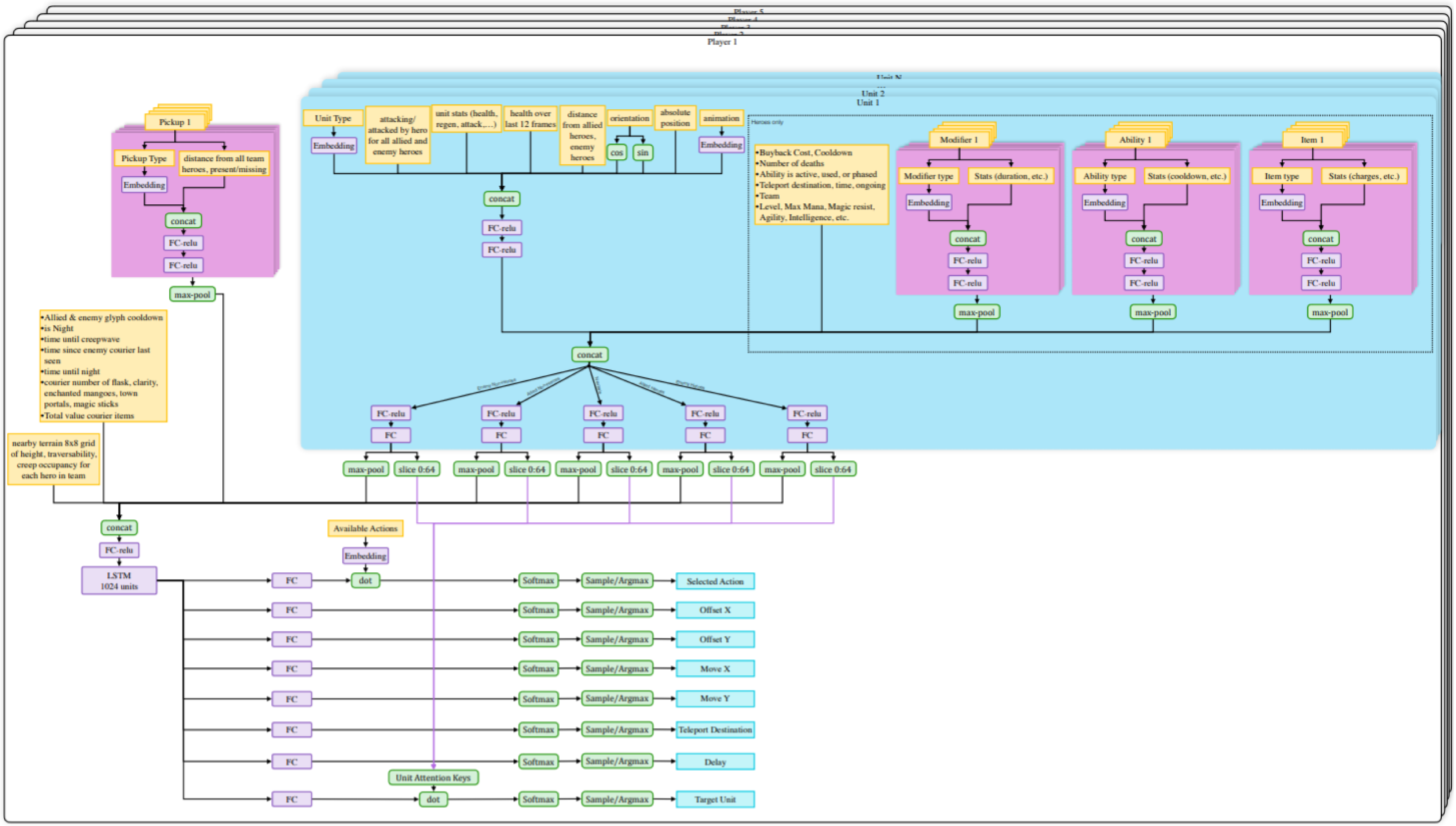
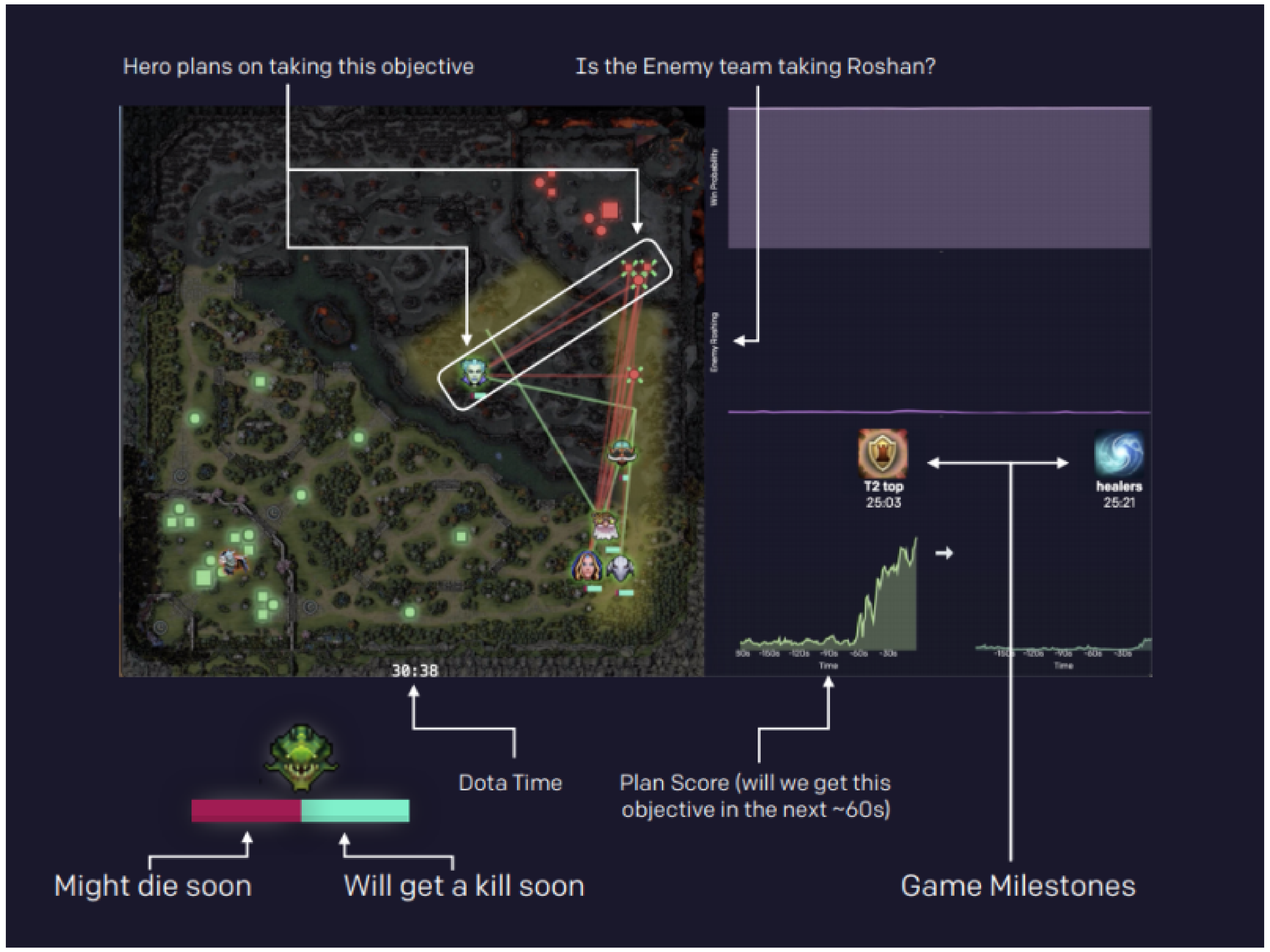
OpenAI Five: Dota 2



https://d4mucfpksywv.cloudfront.net/research-covers/openai-five/network-architecture.pdf

OpenAI Five: Dota 2

- The agents are trained by **self-play**. Each worker plays against:
  - the current version of the network 80% of the time.
  - an older version of the network 20% of the time.
- Reward is hand-designed using human heuristics:
  - net worth, kills, deaths, assists, last hits…



- The discount factor γ is annealed from 0.998 (valuing future rewards with a half-life of 46 seconds) to 0.9997 (valuing future rewards with a half-life of five minutes).
- Coordinating all the resources (CPU, GPU) is actually the main difficulty:
  - Kubernetes, Azure, and GCP backends for Rapid, TensorBoard, Sentry and Grafana for monitoring…

1 - TRPO: Trust Region Policy Optimization (skipped)

Trust Region Policy Optimization

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Schulman et al. (2015) Trust Region Policy Optimization. 1889–1897. http://proceedings.mlr.press/v37/schulman15.html.

TRPO: Trust Region Policy Optimization

- We want to maximize the expected return of a policy π_θ, which is equivalent to the Q-value of every state-action pair visited by the policy:

$$\mathcal{J}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [Q^{\pi_\theta} (s, a)]$$

- Let's note θ_old the current value of the parameters of the policy π_{θ_old}.
- Kakade and Langford (2002) have shown that the expected return of a policy π_θ is linked to the expected return of the current policy π_{θ_old} with:

$$\mathcal{J}(\theta) = \mathcal{J}(\theta_\text{old}) + \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}} (s, a)]$$

where

$$A^{\pi_{\theta_\text{old}}} (s, a) = Q_\theta(s, a) - Q_{\theta_\text{old}} (s, a)$$

is the **advantage** of taking the action (s, a) and thereafter following π_θ, compared to following the current policy π_{θ_old}.

- The return under any policy θ is equal to the return under θ_old, plus how the newly chosen actions in the rest of the trajectory improves (or worsens) the returns.

Kakade and Langford (2002) Approximately Optimal Approximate Reinforcement Learning. Proc. 19th International Conference on Machine Learning, 267–274.

TRPO: Trust Region Policy Optimization

- If we can estimate the advantages and maximize them, we can find a new policy π_θ with a higher return than the current one.

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}}(s, a)]$$

- By definition, $\mathcal{L}(\theta_\text{old}) = 0$, so the policy maximizing $\mathcal{L}(\theta)$ has positive advantages and is better than π_{θ_old}.

$$\theta_\text{new} = \text{argmax}_\theta \; \mathcal{L}(\theta) \; \Rightarrow \; \mathcal{J}(\theta_\text{new}) \geq \mathcal{J}(\theta_\text{old})$$

- Maximizing the advantages ensures **monotonic improvement**: the new policy is always better than the previous one. Policy collapse is not possible!
- The problem is that we have to take samples (s, a) from π_θ: we do not know it yet, as it is what we search. The only policy at our disposal to estimate the advantages is the current policy π_{θ_old}.
- We could use **importance sampling** to sample from π_{θ_old}, but it would introduce a lot of variance:

$$\mathcal{L}(\theta) = \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_{\theta_\text{old}}} [\frac{\pi_{\theta}(s, a)}{\pi_{\theta_\text{old}}(s, a)} \, A^{\pi_{\theta_\text{old}}}(s, a)]$$

TRPO: Trust Region Policy Optimization

- In TRPO, we are adding a **constraint** instead:
  - the new policy π_{θ_new} should not be (very) different from π_{θ_old}.
  - the importance sampling weight $\frac{\pi_{\theta_\text{new}}(s,a)}{\pi_{\theta_\text{old}}(s,a)}$ will not be very different from 1, so we can omit it.
- Let's define a new objective function $\mathcal{J}_{\theta_\text{old}}(\theta)$:

$$\mathcal{J}_{\theta_\text{old}}(\theta) = \mathcal{J}(\theta_\text{old}) + \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_\theta}[A^{\pi_{\theta_\text{old}}}(s, a)]$$

- The only difference with $\mathcal{J}(\theta)$ is that the visited states s are now sampled by the current policy π_{θ_old}.
- This makes the expectation tractable: we know how to visit the states, but we compute the advantage of actions taken by the new policy in those states.

TRPO: Trust Region Policy Optimization

- Previous objective function:

$$\mathcal{J}(\theta) = \mathcal{J}(\theta_\text{old}) + \mathbb{E}_{s \sim \rho_\theta, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}} (s, a)]$$

- New objective function:

$$\mathcal{J}_{\theta_\text{old}} (\theta) = \mathcal{J}(\theta_\text{old}) + \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}} (s, a)]$$

- It is "easy" to observe that the new objective function has the same value in θ_old:

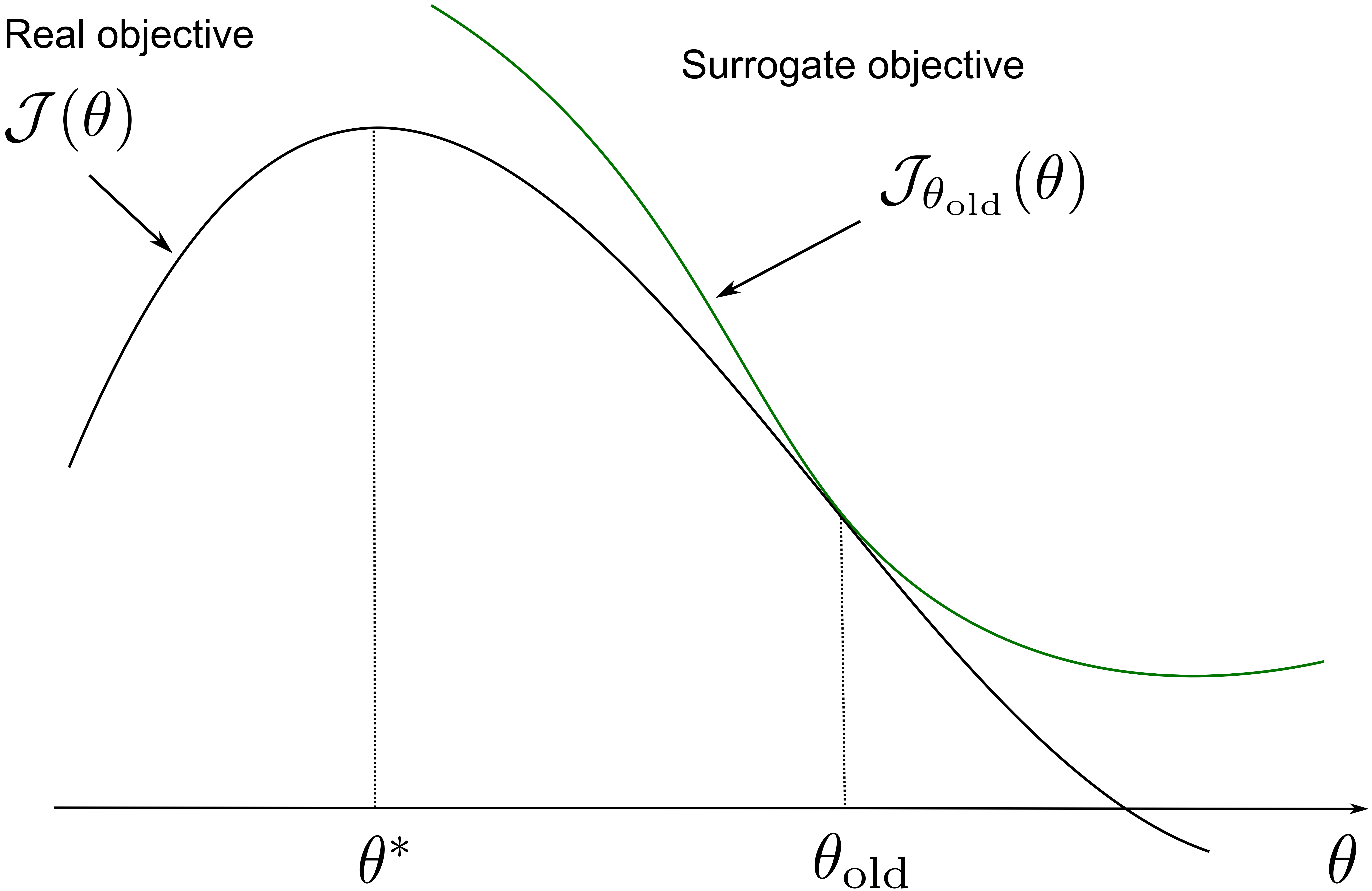
$$\mathcal{J}_{\theta_\text{old}} (\theta_\text{old}) = \mathcal{J}(\theta_\text{old})$$

and that its gradient w.r.t. θ is the same in θ_old:

$$\nabla_\theta \mathcal{J}_{\theta_\text{old}} (\theta)|_{\theta = \theta_\text{old}} = \nabla_\theta \, \mathcal{J}(\theta)|_{\theta = \theta_\text{old}}$$

- At least locally, maximizing $\mathcal{J}_{\theta_\text{old}} (\theta)$ is exactly the same as maximizing $\mathcal{J}(\theta)$.
- $\mathcal{J}_{\theta_\text{old}} (\theta)$ is called a **surrogate objective function**: it is not what we want to maximize, but it leads to the same result locally.

TRPO: Trust Region Policy Optimization



TRPO: Trust Region Policy Optimization

- How big a step can we take when maximizing $\mathcal{J}_{\theta_\text{old}}(\theta)$? π_θ and π_{θ_old} must be close from each other for the approximation to stand.
- The first variant explored in the TRPO paper is a **constrained optimization** approach (Lagrange optimization):

$$\max_\theta \mathcal{J}_{\theta_\text{old}}(\theta) = \mathcal{J}(\theta_\text{old}) + \mathbb{E}_{s \sim \rho_{\theta_\text{old}}, a \sim \pi_\theta} [A^{\pi_{\theta_\text{old}}} (s, a)]$$

$$\text{such that: } D_\text{KL}(\pi_{\theta_\text{old}} || \pi_\theta) \leq \delta$$

- The KL divergence between the distributions π_{θ_old} and π_θ must be below a threshold δ.
- This version of TRPO uses a **hard constraint**:
  - We search for a policy π_θ that maximizes the expected return while staying within the **trust region** around π_{θ_old}.

TRPO: Trust Region Policy Optimization

- The second approach **regularizes** the objective function with the KL divergence:

$$\max_\theta \mathcal{L}(\theta) = \mathcal{J}_{\theta_\text{old}}(\theta) - C \, D_\text{KL}(\pi_{\theta_\text{old}} || \pi_\theta)$$

where C is a regularization parameter controlling the importance of the **soft constraint**.

- This **surrogate objective function** is a **lower bound** of the initial objective $\mathcal{J}(\theta)$:

1. The two objectives have the same value in θ_old:

$$\mathcal{L}(\theta_\text{old}) = \mathcal{J}_{\theta_\text{old}}(\theta_\text{old}) - C \, D_{KL}(\pi_{\theta_\text{old}} || \pi_{\theta_\text{old}}) = \mathcal{J}(\theta_\text{old})$$

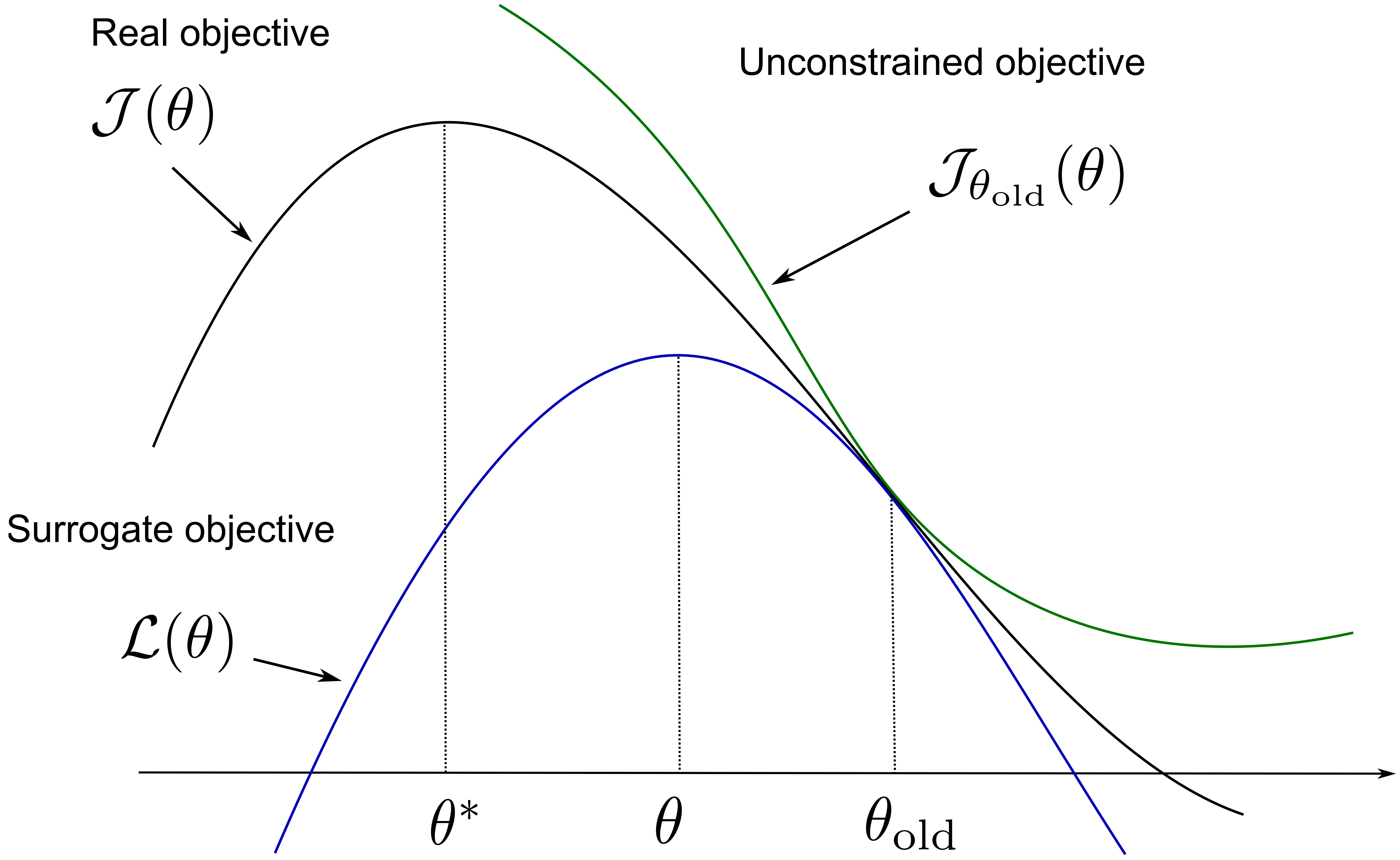
2. Their gradient w.r.t θ are the same in θ_old:

$$\nabla_\theta \mathcal{L}(\theta)|_{\theta = \theta_\text{old}} = \nabla_\theta \mathcal{J}(\theta)|_{\theta = \theta_\text{old}}$$

3. The surrogate objective is always smaller than the real objective, as the KL divergence is positive:

$$\mathcal{J}(\theta) \geq \mathcal{J}_{\theta_\text{old}}(\theta) - C \, D_{KL}(\pi_{\theta_\text{old}} || \pi_\theta)$$

TRPO: Trust Region Policy Optimization



TRPO: Trust Region Policy Optimization

- The policy π_θ maximizing the surrogate objective $\mathcal{L}(\theta) = \mathcal{J}_{\theta_\text{old}}(\theta) - C \, D_\text{KL}(\pi_{\theta_\text{old}} || \pi_\theta)$:

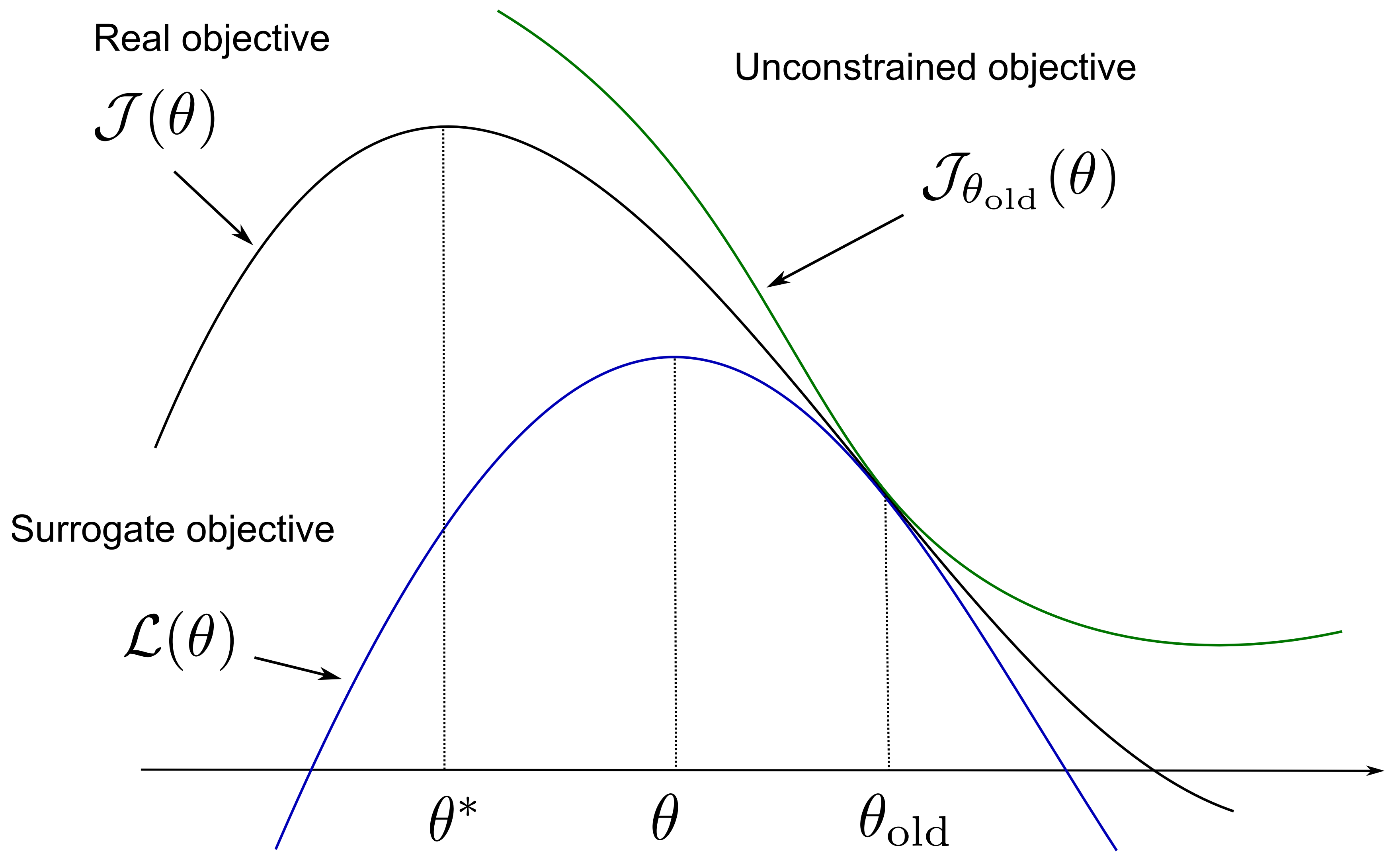
1. has a higher expected return than π_{θ_old}:

$$\mathcal{J}(\theta) > \mathcal{J}(\theta_\text{old})$$

2. is very close to π_{θ_old}:

$$D_\text{KL}(\pi_{\theta_\text{old}} || \pi_\theta) \approx 0$$

3. but the parameters θ are much closer to the optimal parameters θ^*.



- The version with a soft constraint necessitates a prohibitively small learning rate in practice.
- The implementation of TRPO uses the hard constraint with Lagrange optimization, what necessitates using conjugate gradients optimization, the Fisher Information matrix and natural gradients: very complex to implement…
- However, there is a **monotonic improvement guarantee**: the successive policies can only get better over time, no policy collapse! This is the major advantage of TRPO compared to the other methods: it always works, although very slowly.

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